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EDITED BY

W. J. GREENSTREET, M.A.

WITH THE CO-OPERATION OF

F. S. MACAULAY, M.A., D.Sc.

AND

PROF. E. T. WHITTAKER, M.A., F.R.S.

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VOL. X. OCTOBER, 1921. No. 154.

MATHEMATICIANS AND THEIR WORK.*

BY L. J. MORDELL,

WONDERFUL progress has been made in mathematics during the last thirty years, which have been so productive of memoirs, treatises, journals, encyclopedias, societies, and international congresses, that even an expert is in considerable danger of being overwhelmed by the never-ending torrent. The flow is indeed so great that every one must feel how little he knows, and regret that neither the time nor the energy at his disposal will allow him to explore or even to obtain second-hand information of many of the interesting subjects that are brought to his notice.

There is, however, an important region which has not felt the effects of this torrent—one to which the thoughts of many persons must frequently have turned in vain, namely, some introductory account of recent developments and mathematicians.

A start in supplying this deficiency has at last been made by Prof. Cajori, the holder, I think, of the only Chair of Mathematical History, who has brought out a revised and enlarged edition of his well-known *History of Mathematics*. Great changes have taken place since the first edition was published in 1894, and many links connecting this period with the past have been severed. Cayley, Sylvester, Lie, and Weierstrass died a few years afterwards; Stokes, Hermite, Cremona, and Lord Kelvin had some ten years before them, while G. W. Hill, Sylow, and Sir George Darwin were to live for twenty more years. Poincaré, although he did not attain a ripe old age, justified the opinion that he was the greatest mathematician of his time, while new stars, such as Hilbert and Klein, have been shining steadily for many years.

A great wave of mathematical interest and reform has swept the world, the former more especially in America, Italy and Japan; the latter particularly in England, where the spirit of reform has effected many changes in the Mathematical Tripos—the training school of most English mathematicians—and has also resulted in a more intimate knowledge of the progress made in other countries. In France and Germany, which have won renown long since, the changes are not perhaps so striking. Thus the Germans still maintain their supremacy in assembling at some of their universities a galaxy of mathematicians who initiate new ideas, and find without difficulty students of extraordinary brilliance to develop and extend them.

* On *A History of Mathematics*. By Prof. Cajori, Professor of History of Mathematics in the University of California. 2nd edition, revised and enlarged. $4. 1919. (New York: The Macmillan Company. London: Macmillan & Co. Ltd.)

Prof. Cajori has taken the opportunity of adding a few new chapters, and of completely revising, in the light of modern knowledge, that part of his *History* dealing with the older work. The greatest attraction, however, of the volume is the account of the nineteenth and twentieth centuries, which now forms some two hundred of the present five hundred pages, making it almost a new history.

There are two obvious difficulties in writing this section ; not only are many of those mentioned still living, but of far more importance is the difficulty of keeping in touch with, and being conversant with, the relative importance of developments along so many different and unconnected lines. Prof. Cajori has solved the second difficulty by quoting extracts from reports and addresses by numerous experts. These, moreover, he has blended so well with his other material that he has produced a most useful and fascinating volume.

He gives interesting accounts of the people concerned, and also of the subjects that have been dealt with by them, such as, to mention only a few, Analysis Situs, Definition of a Curve, Fundamental Postulates, Theory of Groups, Solution of Numerical Equations, Integral Equations, Problem of Three Bodies, Relativity and Nomography. Very useful, indeed, should they prove in giving one a glimpse of what would otherwise frequently remain unknown.

The reader will find much in this history not only to instruct him and give him a proper sense of proportion, but also to encourage him, as well as to answer numerous questions that suggest themselves. He will notice that most of the standard text-books, *e.g.* Darboux, Picard, Weber, Klein and Fricke, are mentioned, and it is indeed difficult to exaggerate either their importance or the valuable services of their authors. He will see that research is not altogether an easy matter, and that he should not feel discouraged because of efforts apparently in vain ; that many a paper often gives little indication of the enormous amount of preparation and effort required, any more than a plant may give indication of the size of its roots, or an iceberg of its submerged parts. It will occur to him that it is not altogether presumptuous to attempt problems that still remain unsolved, despite the efforts of those gone by, as new methods are being invented of which those in the past could never have any idea.

He will be consoled to find that geniuses such as Galois and Hermite had a lofty contempt for examinations when they had to take them—and that Sir George Darwin and Sylvester found difficulty in reading the papers of others ; that many mathematicians have occasionally made serious errors ; that some of them disliked geometry as much as others disliked analysis.

Mathematics is one of those games in which the onlooker often not only sees very little of the interesting play, but is frequently at a loss to know what is going on. Often the importance of some research depends, not so much upon itself, as upon the developments to which it gives rise. The great importance of a symbol ; of presenting results in such a form as to suggest or to lead to the proof of extremely difficult and general ones ; the fascination of some of those regions referred to as the despair of the mathematician ; the amazing discoveries of the period in the relation of some of the most unexpected subjects to each other, can only be appreciated by the investigator and those who follow in his footsteps.

Who, for example, could have foreseen that questions such as :

1. The law of quadratic reciprocity associated for all time with the names of Euler, Legendre, and Gauss ;

2. The theorem that the roots of an Abelian equation $ax^n + bx^{n-1} + \ldots + k = 0$, where a, b etc., are integers, can be rationally expressed in terms of roots of unity, as stated by Kronecker and proved by Weber and Hilbert ;

3. The complex multiplication of elliptic functions of which the foundations were laid by Abel and Galois ;

4. The theory of algebraic functions and their integrals as developed by Riemann and Weierstrass;

5. Automorphic functions of n variables initiated by Picard;

are so related that the study of the first question and its obvious extensions should lead to all the others?

While discoveries of this kind remain known for a long time only to the expert, a large number of our treasures, thanks to Prof. Cajori, are now on view to the public, for many of whom they would otherwise be inaccessible.

Manchester College of Technology. L. J. MORDELL.

GLEANINGS FAR AND NEAR.

80. Mnemonic for π.—"But I must a while endeavour to reckon right the ratios."—W. F. S. in *Saturday Westminster Gazette.*

81. Mnemonic for π^{-1}.—"O! how I remember you: O difficult equation." (O. 3183098).—E. Leedham, (Buenos Ayres).

82. Praeclarissimus Liber Elementorum Euclidis perspicacissimi. . . . Erhardus Ratdolt Augustensis Impressor solertissimus. Venetiis impressit. Anno Salutis, 1482. 137 ff.

In the MS. Room of the Royal Irish Academy, 24 E. 24, there are several MS. fly-leaves at the beginning and end with mathematical and genealogical notes, the latter partly in Irish, with some Irish verses, by a certain Francis Murphy, A.D. 1785. Afterwards the book belonged to Marcus Cronin, of Tralee (1801), who has scribbled in some curious memoranda, including some amatory lines, written in a very transparent cypher, which are not remarkable for their good taste; these are followed by the goliardic lines, "Est mihi propositum in taberna mori," etc., and at the bottom of the page is the young lady's signature. This is perhaps a unique instance of an edition of Euclid being employed for so mundane a purpose.

On the fly-leaf at the beginning is the inscription: "A gift from a farmer of the County of Kerry, anno 1838, to E. F. Day." The Kerry farmer was perhaps Mr. Marcus Cronin, of Tralee. In the margins of the book are many mathematical notes and genealogical accounts of great Irish families.—M. Esposito, *N. & Q.*, XII. ii. 247.

83. Edw. Montagu.—The husband of Elizabeth Montagu, the famous blue-stocking, was Edward Montagu, "a profound mathematician," says Elizabeth's great-great-niece, Miss Emily J. Climenson, and "a friend of Emerson and other learned men of that day. He was grandson of Charles II.'s Lord High Admiral, Sandwich, and was M.P. for Huntingdon. Mrs. Carter, another Elizabeth, [contributor to *The Rambler*, the friend of Johnson, to whom he wrote, "I am, with respect, which I neither owe nor pay to any other, madam," and again, of whom he said, May 15, 1784, "I dined yesterday at Mrs. Garrick's, with Mrs. Carter, Miss Hannah More, and Fanny Burney. Three such women are not to be found: I know not where I could find a fourth, except Mrs. Lennox, who is superior to them all"], considered Montagu "a man of sense, a scholar, and a mathematician." On October 12, 1746 (four years after his marriage) we find Montagu writing to his wife: "I have with me . . . Mr. Emerson; and amid all these avocations I have found time to study and profit by the Hurworth philosopher as much as I proposed, and shall not, when I return from Newcastle, have occasion to delay my journey for any further instruction from him." At Elizabeth's desire Beattie, the poet, came to converse upon Christianity with Montagu on his death-bed, "but to her great concern," says Beattie, "he set too much value on mathematical evidence, and piqued himself too much on his knowledge in that science."—*Life of Beattie*, iii. 162-3.

MATHEMATICAL NOTES.

584. [K[1]. 13. c.] *A proof of the formula for the volume of a tetrahedron, in terms of the rectangular coordinates of its vertices.*

In most of the text-books of analytical geometry of three dimensions, a proof of this formula is given which has always appeared to me as being somewhat cumbrous, involving as it does a square root which disappears in the result.

Several proofs not involving the square root have occurred to me. Of these, the following is perhaps the simplest.

The equation
$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0, \qquad \text{(i)}$$
being of the 1st degree in x, y, z, and obviously satisfied by substituting (x_1, y_1, z_1) for (x, y, z), is the equation of a plane passing through P_1, the point whose coordinates are (x_1, y_1, z_1). Similarly it passes through the points P_2 and P_3.

Let P_0 be any other point, and P_0' (with coordinates x_0, y_0, z_0') the point where the straight line through P_0, parallel to OZ, meets the plane $P_1P_2P_3$. Hence (i) is satisfied by x_0, y_0, z_0'; so that
$$\begin{vmatrix} x_0, & y_0, & z_0 + z_0' - z_0, & 1 \\ x_1, & y_1, & z_1, & 1 \\ x_2, & y_2, & z_2, & 1 \\ x_3, & y_3, & z_3, & 1 \end{vmatrix} = 0.$$
Expanding this, we get
$$\begin{vmatrix} x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} + \begin{vmatrix} x_0 & y_0 & z_0' - z_0 & 1 \\ x_1 & y_1 & 0 & 1 \\ x_2 & y_2 & 0 & 1 \\ x_3 & y_3 & 0 & 1 \end{vmatrix},$$
or,
$$\begin{vmatrix} x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = (z_0 - z_0') \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \qquad \text{(ii)}$$
$$= P_0'P_0 \times 2 \times \text{Area of projection of triangle } P_1P_2P_3 \text{ on plane } z=0$$
$$= 6 \times \text{Volume of Tetrahedron } P_0P_1P_2P_3.$$

The last step assumes the geometrical proposition that the volume of $P_0P_1P_2P_3$ is the same as that of a tetrahedron whose base is the projection of the triangle mentioned, and whose height is equal to $P_0'P_0$. The proof of this proposition can be made to depend on the fundamental theorem that tetrahedra on a given base have their volumes in the same ratio as their heights. For we can deduce at once the theorem that if one edge of a tetrahedron is shifted along its line and kept unaltered in length, while the opposite edge remains fixed, the volume will be unaltered; and of course the fundamental theorem implies that if one face be fixed, the opposite vertex may be shifted parallel to it, without altering the volume.

Now let M_1, M_2, M_3 be the projections of P_1, P_2, P_3 on the plane $z=0$. Let

P_1P' have the same magnitude and direction as $P_0'P_0$.
M_1P'' " " " " "

Then, by what precedes, the following tetrahedra will have equal volumes:
$$P_0P_1P_2P_3, \quad P'P_1P_2P_3, \quad P''M_1P_2P_3, \quad P''M_1M_2P_3, \quad P''M_1M_2M_3.$$
The last has base $M_1M_2M_3$ and height equal to $P_0'P$.

It is clear from the nature of the proof that the value of

$$\tfrac{1}{6}\times\begin{vmatrix} x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} \text{ will be positive if } z_0 - z_0'$$

and $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ have the same sign, *i.e.* if the circuit $P_1P_2P_3$ as seen from P_0 appears to have positive sense.

The method of proving the volume-formula above explained can be modified to prove the formula for the triangle in two dimensions, and also, by induction, to prove the corresponding formula for the content of a "simplex" in n dimensions.

A modification of the preceding proof, suggested to me by a remark of a friend to whom I showed the form just given, is as follows:

As before, we prove equation (ii).

We deduce that if $P_1P_2P_3$ be fixed and P_0 varies, the determinant on the left is proportional to $(z_0 - z_0')$, and therefore to the volume of the tetrahedron $P_0P_1P_2P_3$. The same is true if any one of the vertices varies, the others remaining fixed.

Consider then successively the tetrahedra $P_0P_1P_2P_3$, $OP_1P_2P_3$, OAP_2P_3, $OABP_3$, $OABC$, where OA, OB, OC are unit lengths measured along OX, OY, OZ.

Corresponding to these we have the determinants

$$\begin{vmatrix} x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}, \quad \begin{vmatrix} 0 & 0 & 0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}, \quad \begin{vmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}, \quad \begin{vmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix}, \quad \begin{vmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}.$$

From the proportionality above proved, we have, *ex aequali*,

$$\text{Vol } P_0P_1P_2P_3 : \text{Vol } OABC = \begin{vmatrix} x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} : \begin{vmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}.$$

Hence, since Vol $OABC = -\tfrac{1}{6}$, and since the last determinant $= -1$, the required result follows. R. F. MUIRHEAD.

585. [K¹. 7. V. 7.]

A MSS. of Fermat, containing the discussion of two porisms, was found amon· Pascal's papers, and is printed in the collected edition of Pascal's works The first porism, which Fermat attributes to Euclid, is enunciated thus: "Datis positione duabus rectis ABE, YBC, sese in puncto B secantibus: datis etiam punctis A et D in recta ABE: quaeruntur duo puncta, exempli gratiâ O et N, à quibus si ad quodlibet rectae YBC punctum, ut H, recta OHN inflectatur, rectam ABD in punctis I et V secans, rectangulum sub AI in DV aequetur spatio dato, videlicet rectangulo sub AB in BD?"

In modern geometry the points I and V would be considered as corresponding points of a homographic set. The given porism thus suggests a further one. R being any point on a circle, to find two fixed points P and Q, such that if RP and RQ meet a given straight line in A and B respectively; then A and B shall be corresponding points of a homographic set. I find that the necessary condition is that P and Q should be on the circle, but otherwise unrestricted. The property of the circle thus foreshadowed is readily deduced from known propositions. Further, we may draw lines through P and Q parallel to the given one, to meet the circle respectively in L and M. Let MP and QL meet the given line in I and J respectively, and let $IJ = 2a$. Then if b be the distance of the centre of the circle from the given line, and r its radius,

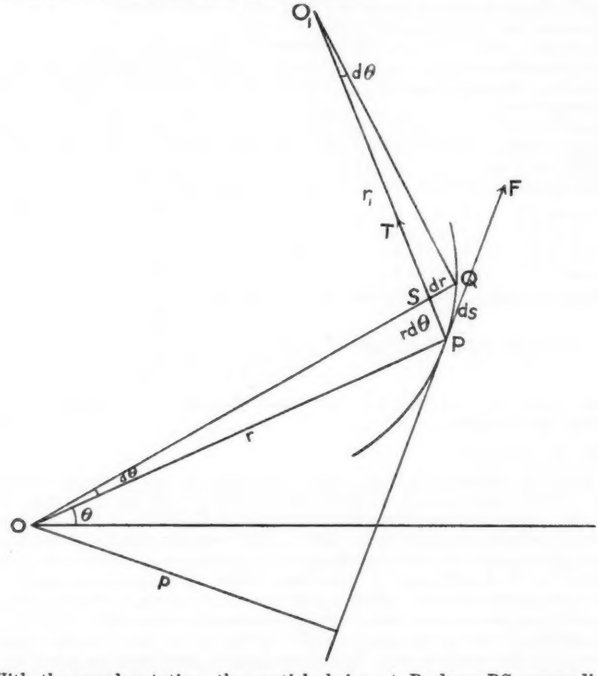
$$AI \,.\, JB = a^2 + b^2 - r^2.$$

J. BRILL.

586. [R. 7. a. α.] *Note on Radial and Transverse Accelerations.*

As soon as a pupil has learned that the deflective acceleration of a particle moving with constant speed in a circle is v^2/r, we may utilise this fact to show that when a particle is moving in a circle with acceleration along its path the resultant acceleration is compounded of v^2/r in the direction of the radius and d^2s/dt^2 in the direction of the instantaneous tangent.

But, if we can, we should go further and deduce the radial and transverse accelerations of a particle moving in any curve. This work is usually reserved for treatises on the Dynamics of a Particle, wherein the results are generally established either by a lengthy discussion of limits of the kind which the pupil nowadays encounters in his first steps in Calculus (see Loney, pp. 45, 46, 47) or by differentiation of $x = r\cos\theta$, $y = r\sin\theta$. Between these two extremes I suggest the following middle course.



With the usual notation, the particle being at P, draw PS perpendicular to OP. The sides of the triangle PQS, as Q approaches P, give the differential displacements:

along the curve,	ds	($\rightarrow$ chord PQ);
transverse to r,	$r\,d\theta$	($\angle PSQ \rightarrow \pi/2$);
in the direction OQ,	dr	($\rightarrow$ direction of OP).

Dividing each side by the time-differential dt, the same triangle may now be taken as the triangle of velocities, since the sides remain in the same proportion. The resultant velocity ds/dt has components $r\,d\theta/dt$ transversely and dr/dt in the direction of r.

It is required to find the radial and transverse accelerations.

The total acceleration cannot be in the direction of ds except when the body is moving in a straight line. But whatever the total acceleration may be, we can resolve it in the directions PS, SQ.

In the direction PS (the direction of θ increasing; r constant) the body would have an acceleration $\dfrac{d}{dt}\left(r\dfrac{d\theta}{dt}\right)$ due to increase of velocity in the PS direction; but r is not constant, and we have to take into account the fact that during the same time dt in which we get the acceleration just noted above, we shall have a deflective acceleration due to the velocity dr/dt in the SQ direction.

Draw QO_1 perp. to OQ to meet PS in O_1.

Then $\angle SO_1Q$ is $d\theta$ and $SQ = dr = r_1 d\theta$.

The velocity along SQ being dr/dt, the deflective acceleration towards O_1 is

$$(dr/dt)^2 \div r_1, \quad or \quad (dr/dt)^2 \div (dr/d\theta), \quad or \quad \frac{dr}{dt}\cdot\frac{d\theta}{dt}.$$

Hence the full acceleration in direction $PS = \dfrac{d}{dt}\left(r\dfrac{d\theta}{dt}\right) + \dfrac{dr}{dt}\cdot\dfrac{d\theta}{dt}$

$$= r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\cdot\frac{d\theta}{dt}$$

$$= \frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right).$$

Similarly, the acceleration due to the body's velocity in the r-direction (θ constant) is $\dfrac{d}{dt}\left(\dfrac{dr}{dt}\right)$. But θ is not constant, and there is a deflective acceleration of amount $(r\,d\theta/dt)^2 \div r$ in the direction PO. Hence the full acceleration in the direction OP is $\dfrac{d^2r}{dt^2} - r\left(\dfrac{d\theta}{dt}\right)^2$.

This latter, the "central" acceleration, seems easier to get by the above method than the former, the "transverse acceleration."

By a purely kinetic method we can argue as follows:

Mass m at P has transverse velocity $r\,d\theta/dt$.

Hence line-momentum in direction $PS = mr\,d\theta/dt$.

Then the moment of momentum about $O = mr^2\,d\theta/dt$,

and the time-rate of change of moment of momentum $= \dfrac{d}{dt}(mr^2\,d\theta/dt)$.

This is equal to the component of Torque, say Tr;

$$\therefore\ T = \frac{1}{r}\frac{d}{dt}(mr^2\,d\theta/dt)$$

$$= m\,.\,\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right).$$

But T is force;

$$\therefore \text{ the transverse acceleration} = \frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right).$$

There can be no component of Torque in the r-direction, hence Tr is really the total Torque and is equal to Fp.

There are occasions when "calculus-dodging devices" not only throw more light on the subject, but also demand more of the real thinking which a fatal facility in calculus may enable even the biggest duffer to escape. The above method (which cannot be new) seems to me to give fully as rigid a "proof" as most given in the text-books. But even if it does not entirely satisfy as a proof, it is at any rate satisfying to one's mechanical sense, and by leading directly to the laws of planetary motion it may serve to assist in

lifting the reproach from elementary mechanics of being too scanty in its kinetic content and overburdened in its mathematical form.

Sydney. F. G. BROWN.

587. [I. 2.] *An Old Result in Novel Form.*

Any number N is written down, and beneath it are written its multiples $2N$, $3N$, ... up to $10N$.

Consider the ten figures in any column. There does not appear to be any general law; yet every complete column must contain *either* a 9 or a 0. Prove this.

[In the first column the gaps may be filled by 0's, and the same result holds.]

If the multiples of N up to $100N$ are written down in the same way, then it is certain that any two adjacent columns will, for some multiple, contain either two 9's or two 0's.

	3	8	7	3
	7	7	4	6
1	1	6	1	9
1	5	4	9	2
1	9	3	6	5
2	3	2	3	8
2	7	1	1	1
3	0	9	8	4
3	4	8	5	7
3	8	7	3	0

King's College, Cambridge. H. W. RICHMOND.

588. [I. 1.] *Mnemonic for μ.*

Base ten: best in practical work. Can't evaluate a logarithm? Nothing can be nicer! A constant is clearly needed first. A what's-his-name (Naperian) logarithm I evaluate, employing a simple series. Nothing stiff—nothing. Multiply by an unusually easy and tractable decimal. . . .

"Nothing" = zero. Words (omitting "nothing") *sic*:

"Litera petiti numeri index."

Simple enough, I'm sure. Just count the letters. Accurate? Yes, I rather think, accurate enough. W. HOPE-JONES.

589. [K. 21. b.] *A Trisection.*

It is well known that a straight line of given length "moving in the legs" of a right angle enables us to trisect any given angle. But that this property leads in turn to a further trisection is, I think, less widely recognised.

Briefly, the mid-point of the moving line traces a circle; and if two positions of the line intersect at a point on its circumference—the given length being bisected there in one case only—then the angles at the centre, subtended by the two chords, are respectively as 3 : 1.

If I may invite the reader to draw a figure, a lettered example will make this perhaps rather more clear.

Take the semi-circle ABC (centre O) with diameter CA and ordinate OB, both produced as necessary towards A and B. Let the straight line DE ($=AC$) move in the right angle AOB, D on AC and E on OB. Then its mid-point will lie on the circumference at some point F; and, except in the tangential case, it (produced, if necessary) will cut the arc again at some point G.

Now let the line move to another position $dFfe$, passing through F but with its mid-point at f. Join OF, OG, and Of.

Then, evidently,

angle $FOC = 3$ times the angle fOA,

also angle $GOC = 3$ „ „ FOA,

and therefore angle $FOG = 3$ „ „ fOF,

and these are the angles at the centre subtended by the two chords FG and fF.

The tangential case where both positions coincide and all the points of intersection lie together at H, on the arc half-way between A and B, calls for little remark; except that the fixed point H is in itself worthy of notice, for in all positions

$$\text{angle } FOH = \tfrac{1}{2} \text{ angle } FOG = 3 \,.\, \text{angle } fOH.$$

85 Wilbury Crescent, Hove, Sussex. C. H. CHEPMELL.

REVIEWS.

A First Course in Statistics. By D. CARADOG JONES. Pp. ix + 286. 15s. net. 1921. (Bell & Sons.)

This is an excellent " first course " to place in the hands of a mathematical student who wishes to develop his work on the statistical side or is interested in probability and has an eye to research on the mathematics of the subject. As the book is one of Bell's Mathematical Series (Advanced Section), it is natural that the subject should be approached in this way, but its use will be wider than that indicated, because it will make a good second course for a person doing statistical work in practice if one of the elementary books on the subject has been read first, and it can be used for revision purposes by those teaching the subject who prefer to give one of the well-known existing text-books to their pupils in the first instance.

The book begins with a short reference to the history of the subject, and the author properly attributes much to Quetelet and Galton—it would be hard to attribute too much to them—but in referring to studies in mortality he gives, to our surprise, the names of Knapp and Lexis, but does not mention those Englishmen who from the middle of the eighteenth century did so much to study mortality in a scientific way. The statistical value of that sort of work is often overlooked, and if it received more attention we might have less inaccurate talk about " birth rates " and " death rates." The author of the book under review is not altogether guiltless, as he writes on p. 7, " if we could assume the birth and death rates to remain constant from year to year, and if we could afford to leave migration out of account, the population would be subject to exactly the same law of increase as capital accumulating at compound interest." How can both birth and death rates remain constant from year to year unless we hypothecate a population which is itself constant ? At the end of the short historical note we find a paragraph summarising concisely the logical development of the subject, and then we are led to notes on index numbers, classification and tabulation, and thence to chapters on averages, variability and skewness. Graphical treatment figures prominently in the representation of statistics, and is described carefully, and interpolation is introduced graphically before it is dealt with algebraically. Perhaps the dangers of relying on the eye in statistical work might have been accentuated : graphical representation is the easiest way of " proving " false conclusions to the uninitiated. The first part of the work concludes with two chapters on correlation, which are particularly well done.

The second part of the book introduces probable errors, and then turns to curve-fitting, explains Pearson's system of frequency curves and gives examples of application to actual statistics. The examples are well chosen, and the proper way of testing whether the curves fit the material is set out. No other system of frequency curves is explained, though references to some of the works dealing with other systems are given. The normal curve of error and the corresponding frequency surface are discussed satisfactorily in the final chapters. The book concludes with Appendices on a few mathematical points, on current sources of social statistics, and on tables to aid calculations.

From this summary it will be seen that the range of subject is well chosen, and it may be added that the work is clearly expressed, but—probably because he does not want to overcrowd a first course—the author does not dwell on and hardly indicates the many difficulties that confront a beginner in curve-fitting, nor does he point out to the mathematical student the dangers of statistical inexperience.

The book is well printed, but why do printers who have a nice taste in type use such anaemic " deltas " and " signs of infinity," and why do they put before the letter denoting the criterion for the various types of frequency curves a hyphen, which is confusing because it is indistinguishable from their minus sign ? There are a few errors that will want correction in a second edition, but they are not of the kind to create trouble and need not be enumerated : such things are easily understood and therefore easily forgiven, but the omission of an index in a book of this kind is beyond a reviewer's understanding. W. PALIN ELDERTON.

Plane Algebraic Curves. By H. Hilton, M.A., D.Sc. Pp. xvi, 388. 28s. net. 1920. (Oxford: Clarendon Press.)

Professor Hilton has an exceptional gift for writing mathematical text-books, and this volume illustrates the fact. Indeed, its merits are so many that it must suffice to emphasise a few of them. The scope of the work (considering its size) is remarkably wide, without leading to scrappiness on the one hand or obscurity on the other. Besides indispensable elements we have a discussion of both classes of cubics, which includes all their leading properties; three chapters on quartics; a most interesting and original chapter on derived curves (evolutes, pedals, etc.); and others of even greater novelty that will be referred to presently.

Like his other text-books, this treatise provides a large number of examples. Many of them are worked out in detail, and the real branches drawn approximately to scale. The amount of work that this has involved must be very great, and deserves the recognition of the student; for, whatever logicians may assert, the visible parts of curves are, and probably always will be, those which chiefly interest those who have a truly geometrical form of mind.

One of the most important things in connection with a given curve is the determination of its Plücker numbers (δ, κ, τ, ι). Up to the present time this remains a troublesome problem, though its solution has been shown to be definite and unique. The general proof would be quite outside the range of a book like this; but it is remarkable how nearly the author reaches it. I think we may say that he gives, in different places, all the elements of the proof except a complete discussion of quadratic $1-1$ transformations, and their effect in resolving multiple points. What he does prove is amply sufficient for the curves which he discusses.

Two definitions of *deficiency* are given (pp. 113, 127): the first is

$$\tfrac{1}{2}(n-1)(n-2)-\delta-\kappa.$$

The other depends on the notion of an adjoint curve. On p. 376 we have a proof that two curves with a $1-1$ correspondence have the same deficiency.

Representation of points on a curve by means of a parameter is discussed, not only for rational curves, but also for elliptic cubics and quartics, and simple applications of Abel's theorem are made, though, of course, the theorem itself is not explicitly used.

One of the most interesting chapters is XX., called "Circuits." This deals with the number of real branches of a curve which has a real equation. Here we have Klein's theorem ($n+i+2t=m+k+2d$) with its application to the non-singular quartic; also fundamental theorems due to Harnack and Hilbert.

It is interesting to see how small an amount of invariant-theory the author has to use, and how much he makes it do for him. Here is a great contrast between this book and Salmon's classical treatise. It is a very good thing that Professor Hilton has made no attempt to imitate his great predecessor, but has followed independent lines of his own. With the latest edition of Salmon, the reprint of Frost's "Curve Tracing," and the present work, an English student has materials for studying a most fascinating subject under the best conditions; and it may be hoped that the theory of algebraic curves will now be more frequently taken up at the Universities. When I was an undergraduate, it was a current saying, not devoid of truth, that all a Tripos candidate need know about higher plane curves was that two cubics each passing through eight fixed points had a ninth fixed point in common. Things have changed since then, and probably will still improve as time goes on.

One word more in conclusion. A great deal of ingenious nonsense has been written about the circular points at infinity, even by people who ought to have known better. Professor Hilton does not introduce v. Staudt's theory at all; so the circular points are merely analytical conceptions; but he uses them with great effect, and in a perfectly unobjectional way, because it is clear throughout that any geometrical language that he uses is merely a paraphrase of algebraical results, where the distinction between real and imaginary is irrelevant.

It might have been an advantage if, besides the references in the text, Professor Hilton had given a short list of really first-rate books and memoirs.

G. B. Mathews.

Elementary Dynamics. A text book for engineers. J. L. LANDON. Pp. 245. 10/6 net. 1920. (Cambridge University Press.)

The sub-title led us to expect some special flavour to suit the taste of those who learn Dynamics with a view of the applications; but this book does not strike us as different from a stock of similar treatises, prepared by examiners of Civil Service candidates, on abstract principles dear to a schoolmaster. And wo-betide the wight who dares to reveal an opinion of his own at all opposed to the strict tenets and doctrine of his cram text-book, crambe repetita; Parkinson and Todhunter all over again.

According to the author's experience of teaching elementary dynamics to a student of Engineering, "the majority find considerable difficulty in grasping the fundamental principles on which the subject is based. There are new physical quantities to be understood, and new principles to be accepted, which can only be expressed in terms of these quantities."

But this experience ought not to be met in a class of engineering students, who come to be instructed in mathematical reasoning on the fundamental principles familiar to them already, if they have any preliminary taste for their Science; or if worthy of the name of young engineer and delighting in it as a boy for pastime.

Perry's method would not have met the approval of the author. Perry's class room was a mathematical workshop, provided with carpenter's bench and tools, vice, anvil, smith's forge, hammers large and small, from sledge to carpenter's.

A number of practical problems were thrown by him at the head of the student, with no information or data, and these he was told to look out for himself in an Engineer's Pocket Book, such as Molesworth. No superfluity of data, as deprecated in Mr. Eric Neville's letter, *Math. Gazette*, p. 224.

After some confidence had been gained in obtaining a correct numerical result to a concrete problem on a large outdoor scale, or else imitated on a smaller scale in the workshop, the student was in a position to appreciate some insight into the theory of the principles he had put into application, and to generalise for himself, in his own way and at his own place; resenting the leisurely spoon feeding of the lecturer, talking down to the slowest member of the class.

The old maxim requires to be reversed, *Principiis enim cognitis, extrema intelligetis,—non multo facilius—sed multo diutius.*

It is not the principles but the exercises that give the flavour to an elementary treatise like this. Some effort has been made to introduce an engineering flavour, but there are too many of the old Parkinson-Todhunter type dear to a schoolmaster, who works to the impending examination in view.

And what has an engineer to do with absolute measure? and the C.G.S. system? All this he will unload as soon as he takes to practical work, and the study of an engineering treatise. The Professor should banish the C.G.S. system from the scope of his domain.

Halsey's *Handbook for Draftsmen* is the sort of textbook to put into the hands of the young engineer; the author stigmatizes the C.G.S. system as "a monument of scientific zeal combined with ignorance of practical requirements."

The electrician requires absolute measure for his cosmopolitan calculations, but always in metric units: and he is beginning to discover that he is really using the M.K.S. (metre-kilogram second) system, and not the niggling microscopic C.G.S. units, where a centime-gram is said to weigh 981, nearly a thousand, dynes, and a French penny weighs ten-thousand dynes.

Absolute measure of force should never be employed except with metric units, and preferably in the M.K.S. system; even there it never appears in Statics.

James Thomson was very proud of his invention of that horrible poundal, but it was never of any use except to pass an examination; and there it is a fearful trap for the candidate. Away with the poundal, and scrap the slug too! They have the presumption to suppose they will pass current throughout the Newtonian universe.

We notice the abbreviation for it, pdls, alongside of lbs, and lbs wt on the sq-inch. Academic prudery should shudder at the barbarism of lbs for pounds or ozs too for ounces because lb is the abbreviation of the Latin *libra*, and oz of *uncia*.

And of the millions of pressure gauges at work, familiar by sight to the class of our author, not one is to be found graduated in the language he uses, but all in pounds on the square inch, lb/inch^2; or else in atmospheres of one kilogram per square centimetre, kg/cm^2, of about 15 lb/inch^2.

The Hospitalier notation has not yet reached the Cambridge Engineering School, in expressing derived units, of pressure in lb/ft^2, density in lb/ft^3 or kg/m^3, velocity in f/s, or m/s, acceleration in f/s^2 or m/s^2; $g=32$, f/s^2, or 9·81, m/s^2. It would replace the chapter on Units and Dimensions, very unconvincing to the young engineer; no message to him.

The Professor of Engineering should make a visit of inspection of his class rooms, to see that the lectures are more suitable for the future executive and consulting engineer. The present treatise is a revelation of humble bread and meat standard carefully pruned to small dimensions. Too much time is allowed in the drawing office, the work is too easy and non-intellectual. It is much more valuable for a designer to seize the salient points of a machine and give a scale representation of them in a free pencil or blackboard drawing, or in chalk on the apron of a forge, with a few figured dimensions to work from; this will replace the careful product of the drawing office.

There would be a howl at the suggestion that some historical interest should be inculcated, say in the works of Hero, Archimedes, Archytas; and in the original Greek as required of a Horace at the University of Athens. Bread and meat studies cannot afford the time; not even for a course in the Sublime Calculus; this must be put off till after the degree, meaning to the Greek Kalends, An education is revealed here suitable for a subordinate capacity. But when we look round our great manufacturing establishments, we see the managing director a product of Oxford Greats.

G. Greenhill.

Ueber Kongruenzen von den fünften und höheren Graden nach einem Primzahl-modulus. By A. Arwin. Pp. 46. 8vo. 1918. (Stockholm.)

This memoir is an extension of an earlier one on the solution of congruences of the 3rd and 4th degree to congruences of the 5th and higher orders. Taking

$$F(x)=x^5+ax^4+bx^3+cx^2+dx+f\equiv 0 \pmod p$$

the author distinguishes seven types of this congruence according to the number of linear, 2^{ic}, 3^{ic}, etc., factors contained in $F(x)$, and he shows that these types are determined by the condition

$$x^{p^i}\equiv x (\text{mod } p),\ [i=1,\ 2,\ 3,\ \dots\ 6],\ \text{when } p=10n+1,$$

and gives several examples with $p=11$. The more difficult cases of $p=10n-1$, $10n\pm 3$ are also developed in a general manner, but without examples. The general process of actual solution is detailed at some length, and examples are given for moduli $p=11$ and 41. The most difficult case appears to be when $F(x)$ is an irreducible function, *i.e.* not resolvable into factors (mod p). This case is developed at some length in a manner suited to congruences of degree higher than the 5th. The general problem is a difficult one, and the process of solution is difficult and tedious even with small prime moduli as $p=11$, 41. The memoir would be improved by division into numbered sections.

Die Kongruenzen $(\lambda+1)^p-\lambda^p-1\equiv 0$ (mod p^ν), und die Natur ihrer Lösungen. By A. Arwin. Pp. 38. 4to. 1921. (Leipzig.)

This memoir is an extension to the modulus p^ν of an earlier one on the same congruence to modulus p^2 by the same author. It is divided into seven sections, preceded by a Table of Contents thereof, with a very clear Introduction of two pages, which makes reference to any part quite easy.

The author shows that the existence of a solution $\lambda \equiv a \pmod{p^\nu}$ requires the co-existence of the two congruences

$$\left.\begin{array}{r} a^p \equiv a + m_1 p + m_2 p^2 + \ldots + m_{\nu-1} p^{\nu-1}, \\ (a+1)^p \equiv a + 1 + n_1 p + n_2 p^2 + \ldots + n_{\nu-1} p^{\nu-1} \end{array}\right\} \pmod{p^\nu},$$

together with certain relations among the sets of coefficients m_r, n_r; and he proceeds to show how the m_r can be computed in a fairly simple way, if they are worked out for several bases $a, b, c \ldots$ at same time by the solution of a system of linear congruences (mod p). In § 3 a number of examples are given, among which occurs Meissel's result $2^{1093} \equiv 2 \pmod{1093^2}$ [a kind of result till recently supposed impossible for base 2]. A complete Table of Solutions (a, x_0) of the congruences $a^p \equiv a + x_0 p \pmod{p^2}$ is given on pages 37, 38 for all primes $p < 100$ for all values of $a \not> \frac{1}{2}(p \mp 1)$. It is next shown how the solution of $x^n \equiv +1 \pmod{p^\nu}$ can be found in a nearly similar way. Later sections show the connection between these methods and some of Bachmann's work. ALLAN CUNNINGHAM, Lt.-Col., late R.E.

The Mechanical Principles of the Aeroplane. By S. BRODETSKY. Pp. vi + 272. 21s. net. 1921. (J. & A. Churchill.)

A reader of this book will complain that the author has forgotten the oft-quoted words of "T and T′" that "nothing can be more fatal to progress than a too confident reliance on mathematical symbols," or else that he has ignored them on the ground that he is discussing the *Theory* only of the Mechanical Principles of the Aeroplane.

The serious business of the book begins with an arid chapter on "dimensions," and proceeds to the ordinary theory of resisted particle motion and the Phugoids. Two chapters on the general motion of the aeroplane follow, consisting principally of sets of lengthy and laborious equations encumbered with essential functions of undetermined intricacy. We then have a long orthodox chapter on two-dimensional hydrodynamics.

The two concluding chapters attempt to apply this preceding theory to practical aeroplane problems, but the reader will be disappointed if he hopes to find anything like, say, a detailed analytical discussion of the differences in construction between a sober "artillery bus" and a lively scout.

It is to be trusted that, when a second edition of this book is issued, the author will face seriously the difficult problem of illustrating and criticising the theory by actual flight trials and laboratory tests. There is now an enormous volume of such literature. Space can easily be made by omitting the chapter on hydrodynamics, and by abandoning throughout the book the stultifying attempt to write for the student untrained in the fundamentals of mechanics and mathematical physics. Such a student will make no real progress in reading this work, even as it stands.

The labour of discussing experimental results is certainly extremely heavy, but is surely of intense interest and importance. If the author will only do this, we may yet be grateful to him for a classic. He has the opportunity.

J. P. C.

The Mathematical Theory of Electricity and Magnetism. By J. H. JEANS. Fourth edition. Pp. viii + 627. 24s. net. 1920. (Camb. Univ. Press.)

This treatise, so useful on its first appearance, does not wear well, and it is to be hoped that the weight of the author's great reputation will not be attached to another edition unless the whole work can be recast. The prejudice against using vectorial notation, at least as a species of shorthand, grows weaker every year. Students who can be referred elsewhere for the properties of confocal conicoids and are assumed to have heard of Riemann surfaces are not in need of definitions of the addition and multiplication of complex numbers. To reproduce Helmholtz' deduction of the state of stress in a dielectric without mention of Larmor's criticism is unwise. And it is not "found *as an experimental fact* that in a homogeneous conductor the lines of flow coincide with the lines of force"; it is difficult to imagine the

actual tracing of lines of either kind inside a solid medium, and here, as in every branch of mathematical physics, the evidence for the assumptions underlying the mathematical theory is the agreement of *practicable* observations with calculations and deductions of extreme complexity.

The distinctive feature of the new edition is a chapter on relativity. Here the general explanations are as admirable as we should expect or as we could desire, but perhaps a more enterprising exposition of the mathematical formulae would have been helpful to the readers for whom this book is designed. As usual we are directed to measure the time t by a new variable τ, the product of it by the velocity of light, and then we are invited to "imagine a four-dimensional space constructed in which x, y, z, τ are orthogonal rectilinear coordinates." Is it too much to assert that the one thing we can *not* do with this space is to *imagine* it, and that all the difficulties and paradoxes in the theory arise from attempts to achieve this impossibility? "The axes of x', y', z', τ' can be obtained from those of x, y, z, τ by a pure rotation in the four-dimensional space." "To determine what physical meaning is to be assigned to θ, we notice that the axes of one observer move relative to those of the other with a velocity $-iC \tan\theta$." Sentences such as these can only bewilder a student accustomed to regard Cartesian axes as scaffolding set up in a given space, especially if no account even of three-dimensional space as an algebraic construction has ever been offered to him; what is he to understand by a "pure rotation", on what authority is he to recognise that *in four-dimensional space with complex coordinates* any displacement of a "rigid body" with a fixed point is a "pure rotation", and how is he to attach a *physical meaning* to a complex number θ on the ground that the function $(e^{i\theta}-e^{-i\theta})/(e^{i\theta}+e^{-i\theta})$ has a known value? In asking the questions, we are venting a disappointment that is itself a compliment.

This treatise gives unequalled training in the discussion of problems in the older parts of the mathematical theory of electricity and magnetism by means of Cartesian coordinates, and if we are not contented that in dealing with relativity Mr. Jeans is no worse than other writers that is because we expected him to be much better.

Précis de Calcul Géométrique. By R. Leveugle. Pp. lvi + 400. 1920. (Gauthier-Villars.)

If the word may be used of a prisoner of war, Lt.-Col. Leveugle was fortunate. A number of his companions, inadequately equipped to utilise German works on physics and applied mathematics that were available, turned to him, and his explanations to them of quaternions and of the calculus of extension, illustrated by a multitude of examples worked for their benefit, form the basis of a splendid text-book. The discussion of the calculus of extension does not go beyond geometric interpretations of some of the algebraic formulae, but the vector analysis includes accounts of differentiation and of the Hamiltonian operator ∇, a valuable chapter on the multiple integrals associated with gradients and curls, and applications to differential geometry, to kinematics, to the theory of elasticity, and to the electro-magnetic theory of light.

An exposition of the more elementary of Grassmann's ideas in the lucid prose of which the best French mathematicians are masters is very welcome. If text-books on quaternions are not lacking in our country, we have none better designed or better executed than this, and the author's enthusiasm for Hamilton's own writings is delightful. But it must not be supposed that the book consists of two parts which merely happen to be between the same covers. It is not until products are introduced that the two subjects diverge —is not the vector to Hamilton sometimes the difference between two points? —and the experiment of developing them together in an elementary course has proved in Lt.-Col. Leveugle's hands a triumphant success.

The English reader will be puzzled to identify the "Royal Collège de Cambridge" at which Tait is said to have been "professeur," and will be amused to find the index regarded as a feature sufficiently novel to be ascribed to a definite influence, that of Grassmann; he may wonder from whom the value of a table of contents so copious as to be a summary was

learnt if not from Hamilton. The one serious blemish on the book is the multitude of trivial misprints, but if the work meets with the reception it deserves abroad as well as at home, the author will have an early opportunity of correcting these in a second edition.

Some Famous Problems of the Theory of Numbers and in particular Waring's Problem. By G. H. Hardy. Pp. 35. 1s. 6d. 1920. (Clarendon Press.)

An Introduction to Combinatory Analysis. By P. A. MacMahon. Pp. viii + 71. 7s. 6d. 1920. (Camb. Univ. Press.)

The pure mathematician is often challenged to give to the layman some idea of what he tries to do and how he sets to work, to say why he wishes to answer the questions that he asks, and to justify the devotion of his intellect to studies which do not affect the material welfare of his fellows. The last of these demands is met by Prof. Hardy in a few inimitable sentences that deserve to become classical, but it is the first that the greater part of his inaugural lecture, here in print, was designed to satisfy; in Major MacMahon's booklet, akin to the lecture, because it can be read without previous training in higher mathematics, methods as well as problems are described in detail.

Readers of the *Gazette* need not be told that Prof. Hardy's account of the problems of which he talks is perfect; even an Oxford audience must have understood most of what he said, and we have an uneasy feeling that his lucidity may have proved his undoing. How can his new University respect a Professor who, far from dealing with matters at the limit of human intelligence, announces that one of his deepest desires will be satisfied when he learns exactly how many fourth powers are required for the expression of an arbitrary large whole number? For, of course, it is not the goal but the game that attracts, and when players of ping-pong and old-maid can realise the pleasure that Prof. Hardy finds in tennis and vint, those who have not attacked the problems for themselves will be able to appreciate both the excitement of the investigations of which glimpses are given here and the skill which their pursuit requires.

As a summary of the facts relative to Waring's problem known when it was delivered, Prof. Hardy's lecture is of great interest, and to the reader familiar with Cauchy's theorem and with the idea of a generating function he has succeeded in explaining the nature of his researches so far as to convey an idea of their fascination. On the other hand, while Major MacMahon is doubtless right in claiming that to read his "Introduction" no mathematical equipment is necessary—the novice will have his self-confidence put to a severe test by the misprints in the scheme on p. 12—and while there could not be a better account than this for the mathematician who requires to learn the principles of combinatory analysis, it is to be feared that the normal effect of the book on the outsider will be to deepen his wonder that people should take interest in such things; for the work, well written as it is, has to be read with attention, and the reader who is not constitutionally attracted by the questions discussed will hardly be convinced that the trouble taken would not have brought greater return if expended elsewhere. It is only fair to add that in presenting his subject in an elementary form Major MacMahon's object was not to illustrate to the sceptic the beauties of mathematics, but to discover to the mathematician valuable points of view that are easily overlooked in an elaborate treatment, and that this purpose has been most admirably achieved.

The Absolute Relations of Time and Space. By A. A. Robb. Pp. x + 80. 1921. 5s. (Camb. Univ. Press.)

Dr. Robb's work on space and time has been the subject of vigorous discussion for several years, and this synopsis in which we can appreciate his definitions and postulates without the diversion of skipping his proofs is very welcome. Two positive achievements are beyond dispute. The description of 'conical order' is a great boon to such physicists and philosophers as find it hard to understand that a universal 'now' is neither logically nor psychologically indispensable. And the enumeration of the various species of three-

fold, of plane, and of line, that are present within an 'eventful' fourfold—a fourfold whose absolute is associated with the equation $x^2+y^2+z^2-t^2=0$—is complete and valuable.

It is typical of the author's method that normality (perpendicularity) is defined in different ways for different types of line, the ultimate justification of the common word being that "once we have introduced coordinates, the condition of . . . normality . . . is the same in all cases." Now the importance of the eventful fourfold to physics is obvious, and the discovery that the fascinating properties which the pure mathematician associates with isotropic planes have a real interpretation is surely more significant than the construction of 'planes' with these properties by means of a system of logical postulates. There is nothing natural or inevitable about the association of the four definitions by which the normality of intersecting lines is explained here, and the reader is prompted to ask why this study of the fourfold is presented as a *synthesis*, at the cost of such a definition as "An optical line is said to be normal to itself," rather than as an *analysis*.

The answer is emphatic, in the author's claim to have "succeeded in developing a theory of time and space in terms of the relations of *before* and *after*," and even more explicitly to have constructed "a geometry of time of which spacial geometry forms a part." To such a purpose as this, elegance in details must give way, and the crucial question is whether the claim is good. In the construction of the fourfold whose absolute is associated with the equation $ax^2+by^2+cz^2+dt^2=0$, it is evidently *possible* to introduce the coordinates one by one, and by the time any one coordinate is introduced a corresponding *asymmetrical* relation *has* been assumed, implicitly, or explicitly, whatever the values of a, b, c, d. Has Dr. Robb done more than confuse the issue by dealing explicitly with the asymmetrical relation connected with the coordinate t while the corresponding relations connected with the other coordinates are hidden in the postulates which secure that, in his own words, "all the theorems of ordinary Euclidean geometry hold for a separation threefold"? It seems to me that he has not, but I would gladly be convinced that I am wrong. Meanwhile, a systematic treatise on the eventful fourfold is badly needed, and nobody is as well qualified to write it as Dr. Robb.

E. H. Neville.

Introduction to the Theory of Fourier's Series and Integrals. 2nd ed. By H. S. Carslaw. Pp. xi+323. Price 30s. net. 1921. (Macmillan.)

This book is the first volume of the new edition of the author's book on *Fourier's Series and Integrals and the Mathematical Theory of the Conduction of Heat.* Owing to the advance made in the subject since the first edition appeared in 1906, the work has been completely rewritten and divided into two volumes. The first volume is concerned with Fourier's Series and Integrals, while the second volume will be devoted to the Theory of the Conduction of Heat.

From the title one would expect to be plunged immediately into the theory of Fourier's Series, the usual theory of infinite series in general being assumed, but this is not the case. The aim of the author has not been to produce a book containing all the known properties of Fourier's Series and Trigonometrical Series, but to remove some of the difficulties of the applied mathematician. With this end in view he has devoted nearly two hundred pages to series and integrals in general. This part of the book is somewhat in the nature of the first few chapters of a *Traité d'Analyse.* After a historical introduction there are chapters on rational and irrational numbers, infinite sequences and series, functions of a single variable (with special reference to limits and continuity), Riemann integration, uniform convergence of series, and definite integrals (both ordinary and infinite) which contain an arbitrary parameter.

Fourier's Series are first introduced in Chapter VII. First of all the classical theory of Dirichlet is given, followed by Fejér's Theorem and the beautiful application of Hardy's Convergence Theorem to prove the convergency of Fourier's Series under Dirichlet's conditions. Chapter IX. is devoted to Gibbs's Phenomenon, and the last chapter to Fourier's Integrals, the work of

Pringsheim on Fourier's Integral Theorem being used in this chapter. Finally there is an appendix on practical harmonic analysis. Each chapter is followed by a list of references, while at the end of the book a valuable bibliography is given.

There is little to criticise adversely, but it is surprising that the first term of the series is not given as $\frac{1}{2}a_0$, so as to obviate the necessity of giving the first term its exceptional expression as a definite integral. A few misprints have been noted (pages 54, 193, 196, 275 and 283), but these are obvious and will not cause trouble.

The book is an excellent one, not only as an introduction to the study of Fourier's Series, but also as an introduction to the theory of functions of a real variable. It is well and carefully written, and contains enough to satisfy the needs of most applied mathematicians, while it will give the student of pure mathematics a clear and rigorous conception of the subject before he proceeds to the higher parts of the theory. W. N. B.

Applied Aerodynamics. By L. Bairstow, F.R.S., C.B.E. Pp. ix + 565. 30s. net. 1920. (Longmans, Green & Co., London.)

The Dynamics of the Airplane. By K. P. Williams, Ph.D. Pp. viii + 138. 13s. 6d. net. 1921. (John Wiley & Sons, Inc., New York; Chapman & Hall, Ltd., London.)

Soaring Flight: a simple Mechanical Solution of the Problem. By Lt.-Col. R. de Villamil (late R.E.). Pp. 48. 1s. 6d. net. 1920. (Charles Spon, London.)

Professor Bairstow possesses an unrivalled knowledge of the practical and theoretical methods of aerodynamical research. The excellent and fundamental work carried out in the aeronautics department of the National Physical Laboratory owed an incalculable amount to his initiative, experimental skill and mathematical ability. One therefore approaches this book with the expectation that here we shall find a really authoritative statement of the subject of aeronautics. In this one is not disappointed. The book is a veritable mine of matters aeronautical. Unlike most books on aeronautics, Professor Bairstow's treatise includes all kinds of aircraft, namely aeroplanes, airships and kite balloons. He first describes the various standard forms of aircraft, and then proceeds to give the principles of flight for the aeroplane and for lighter-than-air craft. The practical methods of aerodynamics are explained in detail, and the most important results of wind-channel and other experiments are quoted in tabular form as well as in the form of graphs. An interesting chapter follows on aerial manœuvres, in which are discussed looping, spinning, rolling, and circular and helical flight. We then get chapters on airscrews, fluid motion and dynamical similarity, followed by a detailed account of the prediction and analysis of aeroplane performance. Finally, we have a study of stability, and of the details of the disturbed motion of an aeroplane.

As is well known, the study of the stability of an aeroplane involves the discussion of algebraic equations of the fourth and higher degrees. In order to be able to predict what the motion of a disturbed aeroplane will be it is necessary to solve such equations numerically. In the case of the longitudinal and lateral stability of a symmetrical aeroplane, Professor Bairstow has himself given very good approximate formulae for finding the roots. For accurate work, however, the roots have to be found with some precision, and for this purpose the best method available is that of Graeffe. The present book contains an appendix dealing with this method, which, although devised in 1837, has certainly not received the attention it deserves.

There can be no doubt that Professor Bairstow has produced what will long be the standard book on the subject. It contains a vast amount of information, and sets out the most important methods available for the application of mechanical principles to aeronautical problems.

Readers of the *Mathematical Gazette* will naturally be interested in another aspect of the treatment of aeronautical problems. Many of them teach mechanics at schools and at universities, and they feel that in aeronautics they have an eminently suitable subject for inclusion in their curricula.

Students, young and old, are sure to be attracted by such a subject: it is romantic and interesting, it is a subject in which much has already been achieved, but in which much yet remains to be done. There is a great future in store for aviation, commercially as well as scientifically.

There is thus a growing demand for a treatment of aeroplane dynamics which does not present the formidable features of the mathematical investigations of Bryan's *Stability in Aviation*, of Bairstow's *Applied Aerodynamics*, or of other books that have appeared from time to time, and which cater principally for such readers as can appreciate the difficult mathematics required for a complete discussion of aeronautical problems. Occasionally one meets elementary statements of some cases of aeroplane dynamics, but as a rule these statements do not go very far. A book like that now published by Professor Williams is therefore doubly welcome, since it is an attempt to present the dynamics of the aeroplane in an elementary and yet in a fairly exhaustive manner. The author bases his treatment on lectures delivered by Professor Marchis in Paris in the spring of 1919. Professor Marchis is a well-known exponent of the science of aeronautics, and the present book cannot fail to evoke interest in all readers.

We get first a discussion of the plane surface and the cambered surface from the aerodynamical point of view. A detailed account is then given of straight horizontal flight, including the effect of varying air density due to change in altitude. Descent and ascent are treated next, with special reference to gliding, climbing and the ceiling. Circular flight follows, leading on to circular descent. The aerodynamics of the propeller is presented very clearly. The next chapter deals with performance from the point of view of the ceiling and of radius of action. The last two chapters contain the usual kind of account of the problem of stability, preceded, however, by a good general statement of the meaning of stability and the methods used for securing it in aeroplanes. A brief account of Wilson's method for calculating the effects of gusts is also included. There is an appendix dealing with the transformation of units, finishing up with one or two useful tables, and a table of references to other books.

There are a number of obvious mistakes and misprints, as well as one or two loose statements about dynamical and statical principles. One gets the impression that the book was written and produced in a hurry. This is a pity, because the book is undoubtedly useful to students of the subject. One cannot say that it supplies the want for a really clear, elementary text-book for young students, since portions of it are decidedly difficult, even for advanced students. But all who are interested in aeroplane dynamics will find in it much that is of value in their study or research.

Lt.-Col. de Villamil attempts to explain a mystery of bird-flight. Many have made similar ventures in the past, and the present solution does not take us much further. The problem is how a vulture can propel itself through the air, and even soar, without moving its wings, *i.e.* without any propulsive exertion. The author uses the method of "energetics," which, he appears to imagine is a more powerful weapon than "pure dynamics." We need only point out where the fallacy lies. It is on p. 29, lines 9 *et seqq.*, where the author confuses the velocity of the bird relatively to the wind with its velocity relatively to the air. One must not forget that the lift or sustentation depends upon the motion of the bird *through the air*, and in spite of the bird having turned from flying perpendicularly to the wind to flying with the wind, this motion through the air has not been altered, and therefore no energy is available for carrying out the process envisaged by the author.

S. Brodetsky.

Fermat's Last Theorem—Proofs by Elementary Algebra. By M. Cashmore. 3rd Edition. Pp. 31. 4s. net. 1921. (Bell & Sons.)

In this edition Mr. Cashmore gives two more futile "proofs" of Fermat's Last Theorem, and also sets the stamp of his approval upon an erroneous one published more than a century ago by Peter Barlow. In his first one he states erroneously on page 18 that all the solutions of an equation of the form

$$z^n = ax^2 + by^2$$

are given by
$$x\sqrt{a}+y\sqrt{-b}=(u\sqrt{a}+v\sqrt{-b})^n,$$
$$z=au^2+bv^2.$$

The study of such an equation is by no means so simple a matter as Mr. Cashmore thinks, very abstruse points in the Theory of Numbers being involved. An account of some of these difficulties is given in Chapter II. of my booklet, *Three Lectures on Fermat's Last Theorem.*

The misstatement made in the second proof, page 35, is, to put it in its simplest form, that $x/(Ay+b)$ cannot be an integer if all the quantities are positive integers, b prime to x, and x, y having a common factor d. This follows, he says, because $ay+b$ is prime to d, and so cannot divide x. Mr. Cashmore forgets that x may have other factors than d, which can cancel the denominator $ay+b$.

The fallacy in the proof by Peter Barlow, who, as will be seen from page 27 of my booklet, has not been so completely lost sight of as Mr. Cashmore thinks, is that if n, p, q, and r are integers of which the last three have no common factor while n is positive, then
$$\frac{p^{n-1}}{qr}+\frac{q^{n-1}}{rp}+\frac{r^{n-1}}{pq}$$
is not an integer. Clearly the denominator p can be cancelled by q^n+r^n.

College of Technology,
Manchester.
L. J. Mordell.

Graphs showing the relation between the Object and Image Distances of Thin Lenses and Spherical Mirrors. By M. A. R. Khan, B.Sc. Pp. 8. (Nizam Coll., Hyderabad.)

Cartesian graphs of the four hyperbolas
$$\frac{1}{y}\pm\frac{1}{x}=\frac{1}{f}$$
(f pos. or neg.) are given on separate sheets, roughly 10 in. × 8 in.

Explanations are added to enable a student to see whether an image is or is not real, erect, magnified. The graphs thus form a useful resumé of experimental results with which he should have become acquainted in the Physical Laboratory, especially if in going through them he tries to call up mental pictures of the experiments, whose results are here recorded. The value of the pamphlet might be increased without adding to its bulk if some of the blank spaces were utilised for pointing out (i) the relation $UV=f^2$ when the distances U, V from the focal planes are those of a point on the graph from the asymptotes already dotted in; (ii) the method of rapidly finding any number of points when f is given based on the collinearity of (u, o), (f, f), (o, v); (ii) might be shown as an inset in the unoccupied quadrant.

Perspective. By A. S. Percival. Pp. 42. 4s. 6d. net. 1921. (Longmans, Green & Co.)

The object of this little book of 42 pages, as stated in the preface, is (i) to explain Art School rules concisely and clearly; (ii) to show how such of them as require the use of very distant vanishing points may be replaced by others which do not. We have gone carefully through it, and, in our opinion, the author seems to have achieved his aim. The methods which he calls new will be recognised readily by pupils of the late G. A. Storey, from whom we received our first useful instruction in the elements of the art, and the author who hit upon some of them independently acknowledges his partial indebtedness to the *Theory and Practice of Perspective* written by the artist. The mention of this work will be interesting to some of the older members of the M.A., as it was, we believe, the outcome of a paper which he read to us some thirty years ago in the form of a dialogue between Euclid and Apelles. Mr. Percival shows how a table of tangents and secants can be utilised for obtaining by calculation distances along the H.L. without a geometrical construction. It might be useful in a later edition to add a tangent scale graduated both for angles and tangents, with an explanation of its use for those who prefer construction to calculation. There is a short section on Shadows, which, though confined to

a simple case, is useful and probably sufficient to enable a student to extend the methods used to harder cases. We do not know of any work on the same subject which conveys so much clear and useful information in such a small compass. We notice an apparent defect in Fig. 11. Should not the short thick lines drawn downwards through 1′, 2′, 3′, 4′ be parts of $K'1$, $K'2$, $K'3$, $K'4$? If there is any explanation of their use we have missed it.

Textile Machine Drawing. By T. Woodhouse and A. Brand. Pp. 124. 2s. 6d. (Blackie & Son.)

As far as one not directly connected with Textile Industry can judge this moderately-priced little work seems excellently planned and carefully executed. Advice on the selection and use of instruments and the simple mathematical constructions most often wanted occupy the first four chapters. The fifth explains orthographic projection plan and elevation of simple solids, including that of a "roving frame" being found. The remaining chapters show the application of the previous sections to the execution of drawings of various classes of machines used in Textile Industries. Edward M. Langley.

Education and World Citizenship: An Essay towards a Science of Education. By James Clerk Maxwell Garnett. Pp. x+515. 36s. net. 1921. (Cambridge University Press.)

This is preeminently a large book; not only in mere bulk of reading matter, though even this is considerable, but in the size of the problems dealt with, and in the breadth of the concepts brought to bear upon them.

One of Mr. Garnett's aims is to give a certain measure of precision to educational doctrine by avoiding the figurative language to which educationists are so much given, and by formulating his theories, wherever it is possible, in mathematical terms. He delivers us from metaphor and hands us over to mathematics. Many years ago Professor Adams put us on our guard against the insidious suggestion of proof that lurks in metaphor; and Mr. Garnett has pressed the warning still further. Of all present-day metaphors he regards the "broad foundation" metaphor as the most popular and at the same time the most harmful. To say that we must in the primary and secondary school lay a broad foundation of liberal education is to commit ourselves to the view that educating is building, and to expose ourselves to the evils that arise from putting such a view into practice.

Instead of saying that the mind is like something else, such as a plant, a building or a garden, and then drawing unwarrantable deductions from the analogy, Mr. Garnett prefers studying the mind directly. He follows the method initiated by Professor Spearman, who has tried to prove by mathematical analysis that the mind is made up of two factors, one general and the other specific—one entering, in different degrees, into each particular mental function, the other belonging peculiarly to that function. This factor common to the various activities of the mind has been labelled "general ability," and its measure is known as g. The proof referred to rests upon the correlation coefficients that have been calculated between several types of mental ability. The existence of g is said to be proved if the correlations between each of the mental activities tested and each of the others hold a certain relationship which has been called a "hierarchical order"; and the amount of g is estimated by the method of partial correlation. It is true that the validity of the reasoning has recently been challenged by Professor Thomson of the Armstrong College, and that a controversy on the matter still rages; but Mr. Garnett broadly accepts Professor Spearman's line of argument, and carrying it still further demonstrates the existence of certain factors which are not, like g, common to all specific abilities, but are common to a group of them. These he calls group factors. Dr. Webb had already shown that "persistency of motives," or, as Mr. Garnett prefers calling it, Purpose, or p, was one such group factor; and Mr. Garnett now proves that there is another important group factor to which he gives the name of Cleverness, or c. So far does Mr. Garnett's love of mathematical exactitude carry him that he believes that genius is probably measured by $\epsilon=\sqrt{g^2+c^2}$, a conclusion that is more plausible when we realise what interpretation he

gives to g and c. In his views of g he runs counter to the opinion that at present prevails. Professor Spearman holds that g is a central fund of intellectual energy which is unaffected by training. Mr Garnett, on the other hand, identifies it with Will and regards it as cultivable. By Will he means voluntary attention—the power to force oneself to attend to a thing—and not will acting at long range, or tenacity of purpose. This latter is the other group factor p. He means by Cleverness the aptitude for seeing analogies, for associating by similarity rather than by contiguity. In the slang terminology of America a man with much c would be a "sulphite"; with little, a "bromide." The author certainly scores a point here. It is clear that g alone cannot make an intellectual giant.

In arguing for the cultivability of attention Mr. Garnett is not on such safe ground. He quotes high authorities in support of his view; but what is really needed is experimental evidence. If it could be established experimentally that it is possible to improve by practice one's capacity to pin down a wandering attention to the business in hand—one's capacity to enforce voluntarily the thought directed to a topic—it would be an enormous step forward in the science of education. Meanwhile there is no harm, and probably much good, in holding the opinion as a matter of educational faith.

The greater part of the book is concerned directly or indirectly with the aim of education; and in expounding his views of this aim the author deals in great detail with the organisation of knowledge and of purposes in the mind of the learner; an organisation which, rising through different levels of subordination, should end in one supreme purpose—the Christian purpose—"the advancement of the Kingdom of God in the minds of men."

Having in the first half of the book established, provisionally if not finally, the basal principles of education, Mr. Garnett proceeds in the second half to show how these principles may be applied to a national system of education. The scheme he puts forward is democratic in spirit, and aims at giving everybody the type of education indicated, not by social status, but by mental endowment.

The book as a whole bears the mark of a clear, vigorous and original mind and makes a definite contribution to modern educational theory. P. B. B.

A Concise Geometry. By C. V. Durell. Pp. viii + 319. 5s. net. 1920. (Bell & Sons.)

The primary object of this volume, which deals with Plane Geometry only, is "to supply a large number of easy examples in the belief that the educational value of the subject lies far more in the power to apply the fundamental facts of geometry, and reason from them, than in the ability to reproduce proofs of these facts." It may be questioned whether the examining bodies, in attaching so much importance to the reproduction of proofs, do not mistake their functions. There would be far more elasticity if the development were left to teachers subject to sympathetic criticism by inspectors, while examining authorities dealt mainly with the power to apply the fundamentals and reason from them.

The subject matter is arranged in four groups.

I. Triangles and Parallels.
II. Areas, Pythagoras' Theorem, Inequalities.
III. The Circle.
IV. Proportion and Similar Triangles;

and the book consists of about 200 pages of exercises, followed by about 100 pages of bookwork in a compact form for purposes of revision. For convenience, the exercises are arranged as riders on the four groups in succession, followed by construction exercises, similarly arranged, and then by 50 revision papers. In the first part the particular enunciations, with the corresponding figures, of one or more theorems are printed, without proof, just before a set of exercises on those theorems; and the bookwork is arranged to suit those who are revising for examination purposes. The order of the first few theorems is Euclid I. 13, 14, 15, 4, 16, 27, 29, 32, 26, 5, 6, 8. The introduction of the term *locus* is postponed almost to the end of Group III., and then the

double aspect is not brought out. In consequence of this, some of the proofs are not so concise as they might be, and one, at least (theorem 36), is incomplete. As a collection of exercises the book fulfils its purpose admirably.

In looking over these recently published text-books on Geometry, one cannot fail to notice that in this country we have not moved very far away from Euclid. The trivial changes of sequence are commonplace. It may not be considered inappropriate for a reviewer, who looks for the development of a geometry as serious as Euclid's but arranged on somewhat different lines, to make a few brief observations.

Parallels. The fact to realise is that lines in the same plane which slope at equal corresponding angles with a crossing line cannot meet once without meeting twice. Rotation through 180° round the middle point of AC would make $BACD$ take up the position $FCAE$. Hence the lines EAB and FCD cannot meet once without meeting twice. If it be postulated that there cannot exist two distinct straight lines passing through the same two points, it follows that two such lines as EB and FD cannot meet at all. Such lines may then be called parallel lines, and EB and FD may be described as having the same direction. The postulation of Playfair's Axiom then leads to the

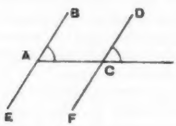


Fig. 1.

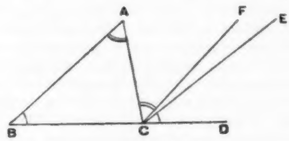


Fig. 2.

converse angle properties of parallels and to the angle-sum property of a triangle;—also to the fact that *If two angles of one triangle are respectively equal to two angles of another triangle, then the third angle of the one is equal to the third angle of the other.* For the sake of the few who may afterwards proceed to the study of non-euclidean geometry, it may be well to handle Euclid I. 32 thus:—A line CE, drawn so that $D\hat{C}E = \hat{B}$, will not meet BA once without meeting it twice; a line CF, drawn so that $A\hat{C}F = \hat{A}$, will not meet BA once without meeting it twice. If it be granted that CE and CF are one and the same straight line, and not otherwise, $A\hat{C}D = \hat{A} + \hat{B}$.

Isosceles Triangles. Let the bisector of the angle BAC of the triangle ABC meet the opposite side BC at D. By folding over AD, AB may be made to lie along AC; but B will not fall on C unless $AB = AC$. When $AB = AC$,

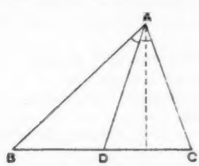


Fig. 3.

however, the two triangles ABD and ACD are congruent. Hence we have the following properties of an isosceles triangle:

(i) *Its base angles are equal.*

(ii) *The bisector of its vertical angle bisects its base.*

(iii) *The bisector of its vertical angle is perpendicular to its base.*

Now, if a perpendicular were drawn from A upon BC it would have to lie

along AD—otherwise, there would be two distinct lines both perpendicular to BC, and therefore parallel, yet meeting at A. Hence:

(iv) *The perpendicular from the vertex of an isosceles triangle upon its base bisects its vertical angle.*

(v) *The perpendicular from the vertex of an isosceles triangle upon its base bisects its base.*

Again, if we join A to the middle point D of the base, we should be drawing the same line as already obtained by bisecting the vertical angle—otherwise, there would be two distinct straight lines both passing through A and D. Hence:

(vi) *The join of the vertex of an isosceles triangle to the middle point of its base bisects its vertical angle.*

(vii) *The join of the vertex of an isosceles triangle to the middle point of its base is perpendicular to its base.*

Again, if we bisect BC at right angles in the plane of the triangle, the line so drawn must pass through A—otherwise, there would be two distinct lines through D, both at right angles to BC in the plane of the triangle. Hence:

(viii) *The perpendicular bisector of the base of an isosceles triangle passes through its vertex.*

(ix) *The perpendicular bisector of the base of an isosceles triangle is equally inclined to the two sides.*

Finally, let DE bisect at D at right angles the base BC of any triangle ABC.

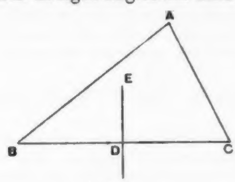


FIG. 4.

By folding over DE, DB may be made to lie along DC and B on C; but BA will not lie along CA unless $\hat{B} = \hat{C}$. So long as the angles B and C are unequal, BA and CA will meet DE at different places. When, however, $\hat{B} = \hat{C}$, BA can be brought to lie along CA by folding over DE; in this case, then, BA and CA meet DE at the same point, and BA would fit on CA. Hence:

(x) *When the base angles of a triangle are equal, the triangle is isosceles.*

The most important point in this connection is to grasp the two facts embodied in the statement. *The locus of a point which moves in a plane in such a way that it is always equidistant from two fixed points in that plane is the perpendicular bisector of the join of the two points.* The applications of this locus are innumerable, and it covers a large and important region of Euclidean Plane Geometry. I venture to suggest that, with the usual sequence, the subject of congruence of triangles occupies too prominent a position, and, compared with the above, is relatively of small importance.

CONGRUENT TRIANGLES. After a preliminary discussion as to the measurements necessary and sufficient to determine a triangle in size and shape, it is realised that one measurement at least must be a measurement of length. In all three cases the triangles ABC and ABD may be placed on opposite sides of a common base AB.

(i) If, in addition to the common side AB,

$$\left\{\begin{array}{l} C\hat{A}B \;\;=\;\; D\hat{A}B \\ \text{side } AC = \text{side } AD \end{array}\right\},$$

by folding over AB, triangle CAB will clearly fit on triangle DAB.

(ii) If two angles of one triangle are respectively equal to two angles of the other, their third angles are also equal—this in addition to the common side AB. Again, by folding over AB, the triangles are clearly congruent.

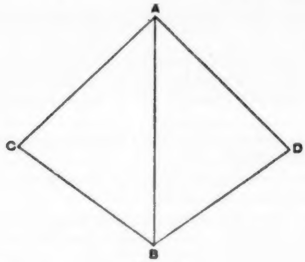


FIG. 5.

(iii) If, in addition to the common side AB,

$$\left.\begin{cases} AC = AD \\ BC = BD \end{cases}\right\},$$

A and B must each be situated in the perpendicular bisector of CD. Hence AB *is* the perpendicular bisector of CD and bisects the angles at A and B. Hence, by folding over AB, the triangles are clearly congruent.

Everyman's Mathematics. By F. W. HARVEY. Pp. 138. 4s. 1920. (Methuen.)

The book contains seven chapters on Arithmetic, Algebra, Logarithms, Geometry, Graphs, Trigonometry, Calculus. Its purpose appears to be to introduce beginners who have had little or no mathematical training to the practical use of elementary mathematics. It is written in simple language, and the various topics are frequently introduced by reference to submarines, aeroplanes and gunnery. But the fog of war has also found its way into the pages, and it seems doubtful whether many students outside the circle of the author's own pupils would find the book intelligible. Here are two specimens taken from the chapter on Geometry: "Two triangles may have their angles equal, but not be the same size or have equal sides." "When two triangles have two sides equal, and the angle between the pairs in each triangle also equal, then the triangles are equal in all respects."

Solid Geometry. By J. W. HENSON. Pp. vii + 88. 3s. net. 1920. (Blackie.)

It is suggested in the preface that this book provides a course in Solid Geometry suitable for classes taking Higher Geometry in senior school examinations and in the Intermediate B.Sc. examinations of London University. There are five chapters dealing with Planes and Straight Lines, Surfaces of Solids, Volumes, Latitude and Longitude, Spherical Triangles, and about 120 exerciess arranged in eight sets. The simplicity of the twelve theorems of Chapter I. is in places more apparent than real. On page 2 the author attempts to prove that two parallels, defined as lines which lie in the same direction, are co-planar. The enunciation of Theorem 3 is "Normals to the same plane are parallel, and conversely, parallel straight lines are perpendicular to the plane, to which one of them is perpendicular." In proving (i), (ii) is quoted; while the proof of (ii) is a mere assertion. Later on, the volume of the right square pyramid forming one-sixth of a cube is given, not as an illustration but as a proof of the usual formula for the volume of a pyramid; and this formula is quoted in proving that similar pyramids are to one another in the ratio of the cubes of their heights or of their corresponding edges.

A First Trigonometry. By WINIFRED WADDELL and D. K. PICKEN. Pp. vii + 78. 1919. (Melville & Mullen, Melbourne.)

A very sound and clearly written text-book. The introductory chapter is intended mainly for the teacher. The seven chapters which follow deal with the Trigonometric Ratios of an acute angle, their graphs, Use of Pythagoras' Theorem, Use of Trigonometric Tables, Practical Applications, Geometrical Applications, Generalisation of Angles and Trigonometric Functions. Those who advocate the early introduction of easy numerical Trigonometry will not find here a text-book suited to their purpose, but it would be most valuable at a later stage to fix ideas and make a sure foundation, before further building. The authors express the projection of AB on a line inclined at an angle C as $\cos C \,.\, AB$, insisting that the numerical factor $\cos C$ should precede AB. Perhaps the expression "£5" ought to be regarded as objectionable.

Analytic Geometry. By MARIA M. ROBERTS and JULIA T. COLPITTS. Pp. x + 245. 7s. 6d. net. 1918. (Chapman & Hall.)

A book for beginners in the use mainly of Cartesian Coordinates. The conspicuous topics are the "equations of loci" and the "loci of equations," and the subject-matter is not confined to algebraic equations of the first and second degrees. The equation of a straight line in the gradient form

$$y - y_1 = m(x - x_1)$$

very properly takes the first and prominent place. The book is overloaded with explanations, instructions and warnings, suggesting that the reader is not expected to do much thinking for himself, and the geometry is somewhat thin. There are chapters dealing with the simplest forms of the equations of the circle, parabola, ellipse, hyperbola; a short chapter on tangents and normals, another on poles and polars introduced by means of the harmonic property, and a very short discussion of the general equation of the second degree. Then follows a chapter on transcendental and parametric equations, and finally a chapter on solid analytic geometry dealing with the plane and straight line, and the forms of surfaces given by the simplest equations of the second degree. The opportunity is not taken to introduce the useful topics associated with "plan" and "elevation," and the polar coordinates of P are defined as OP and its direction-angles with OX, OY, OZ. The book is well illustrated by frequent diagrams, and there is an abundance of easy exercises for the student.

The Analytical Geometry of the Straight-Line and the Circle. By J. MILNE. Pp. xii + 243. 5s. 1919. (Bell & Sons.)

The author confines his attention to Rectangular Cartesian Coordinates, and with these limitations gives a very full treatment of the straight-line and the circle. He is careful to avoid "mere pieces of algebraic work," and maintains throughout a geometrical atmosphere. The diagrams and exercises are very numerous, and the latter are well chosen, many being worked out in detail. In Chapter V. the "polar coordinates" of a point on the circle $x^2 + y^2 = a^2$ are defined as $a\cos\theta$, $a\sin\theta$. The linear expression $lx + my$ is interpreted as twice the area of the quadrilateral (o, o), (m, o), (x, y), (o, l), the figure being drawn with l, m, x, y all positive. Hence the equation $lx + my + n = o$ represents a straight line parallel to the join of (m, o) and (o, l). Later the notion of *gradient* is introduced leading to the equation of a line in the form $y = mx + b$. The form $y - y_1 = m(x - x_1)$ comes in Chap. VIII., and this postponement of the fundamental idea is largely responsible for the unnecessarily slow acquisition of power. There are a few obvious misprints, such as can scarcely be avoided in the first edition of a book of this size,—also a few instances of confusion between theorem and converse. For instance, on page 218 the theorem is enunciated, "Two circles cut orthogonally if the square on the line joining their centres is equal to the sum of the squares on their radii." In the proof which follows, the hypothesis is that the circles cut orthogonally, and the conclusion is that "$C_1C_2^2 = r_1^2 + r_2^2$, as was to be proved."

A School Geometry. By B. A. Howard and J. A. Bingham. Pp. xxvi + 376. 5s. 6d.; or, in two parts, 3s. 3d. each. 1920. (University of London Press.)

This volume covers the course of Euclidean Geometry, Plane and Solid, usually read in schools. In the opening chapters the alternate-angle property of parallel lines is assumed and quoted as "our parallel axiom," and it is claimed that this permits of an order being adopted which is simpler for beginners than the more usual one." But would it not be more natural to assume the equality of corresponding angles? At a later stage (p. 131) Euclid's propositions I. 16, 27, 29 are proved, but the authors fail to point out that they have assumed another parallel axiom instead of that previously adopted, and state that "the alternate angle property of parallels" is now "a proved truth." Altogether there are 100 propositions, 13 being problems. Parts of the book, like the curate's egg, are quite good, but the numerous blemishes are conspicuous and serious. In the preface reference is made to the recommendations of the Board of Education and to the desirability of freeing a boy's mind, by the early introduction of informal work in solid geometry, from "the tyranny of paper." It is difficult, however, to find anything of the nature of solid geometry between the first few pages and the last chapter, which is devoted to "The Elements of Solid Geometry." The six common solids illustrated on page 2 are defined as *regular solids*—"regular because each has a definite shape,"—yet it is proved on page 340 that "there are only five regular solids." On page 12 a *circle* is defined as "a plane figure bounded by one straight line, called its circumference," etc., no mention being made of "the hole in the middle." Later, on page 146, "a circle is the locus of a point moving so as always to be equidistant from a fixed point," the tyranny of paper preventing all mention of the *plane*. On page 4, a *triangle* is defined as "a plane figure bounded by three straight lines," while on page 320 it is proved that "every triangle must be a plane figure." The term *locus* is introduced in Chap. VII., but the double aspect, inclusive and exclusive, though mentioned on page 119, is generally ignored. For instance, on page 118 it is proved that any point situated on the perpendicular bisector of AB is equidistant from A and B, while the unproved converse is afterwards quoted. Again, the statement "Angles in the same segment of a circle are equal" is regarded as equivalent to "The locus of a point at which a fixed straight line subtends a constant angle is an arc of a circle passing through the ends of the line," and later there is an incomplete proof of the theorem, "If one side of a quadrilateral subtends equal angles at the other vertices, the quadrilateral is cyclic."

Though geometrical magnitudes are generally treated as measurable, and, on this assumption the proportion properties of parallels are established on page 72, the subject of similar triangles is postponed to Chap. IX., which opens with an algebraic treatment of ratio. In that chapter it is proved that the areas of triangles with equal altitudes are proportional to their bases by quoting the usual formula for the area of a triangle, and this is done to introduce afresh the proportion properties of parallels in the approved manner of Euclid VI.

On page 331 one plane is proved to be perpendicular to another before the idea of a *dihedral angle* has been introduced and before the conditions of perpendicularity of two planes are discussed. The solid geometry is not illuminated by the use of *plan* and *elevation*.

The book is well printed, the diagrams are neat and clear, there are numerous exercises arranged in sets, the answers to the numerical exercises are given at the end of the book, following an index of definitions and terms, and it is claimed that the volume covers the requirements of the Matriculation and Intermediate Examinations of the University of London.

It is refreshing to find Astronomy in the introductory chapter. "The sun is in the south at noon and is due west at 6 o'clock in the evening. At what time is it south-west? In what directions is it at 1 o'clock and at 4 o'clock respectively?" Messrs. Howard and Bingham give the answers, "3 o'clock; 15° W. of S.; 30° S. of W." In the next exercise the six o'clock limitation is removed, and there is suggested a practical and general application of the

same simple and obvious truth. The boys of Warwick School, freed from the tyranny of paper, are already contemplating an educational and ornamental addition to their playground in the form of a common-sense sun-dial. The style, instead of leaning over in the usual drunken fashion, will stand erect at the centre of a horizontal circle graduated at equal intervals of 15° to show the hours.

It is all so simple; there is no danger of boys "coming to regard geometry as a rather useless system of hair-splitting"; and the "sense of logical reasoning is greatly strengthened." Are not the letters Q.E.D. appended to every theorem?

A Geometry for Schools. Edited by A. CLEMENT JONES. Pp. viii + 96 + viii + 128 + vii + 104. In three parts, 2s., 2s. 6d., 2s. 6d. 1920. (Arnold.)

This book embodies in the form of a text-book the recommendations outlined in the Board of Education circulars, Nos. 711 and 851, as interpreted by the Mathematical Staff of the Bradford Grammar School; and there can be no doubt that the work has been extremely well carried out. Stage I., covering 45 pages, deals with geometrical concepts in a manner suited to young children, appeal being made to observation and intuition with reference both to plane and solid geometry. Stage II., covering 38 pages, postulates Euclid I. 13, 14, 15, 32, 8, 4, 26, 27, 29, appeal being made to intuition and drawing. Stage III., covering 211 pages, gives the formal treatment, mainly on Euclidean lines but with a simplified order, of plane geometry only, based on the nine theorems postulated in Stage III. Then follows an Appendix, designed to accommodate the book to the requirements of certain examinations, by giving formal proofs of Euclid I. 13, 14, 15, 4, 8, 16, 27, 29, 32, 26.

The 90 propositions of Stage III. are arranged in 11 groups, the first group being 9 simple constructions which come at the end of Vol. 1. Vol. 2 comprises groups 2 to 6, and includes Loci, Simple Properties of one Circle, Areas, Similar Figures. Vol. 3 comprises Areas (Introducing Algebraic Proofs), Inequalities, Harder Properties of Circles, Construction of Circles, Concurrent Lines and Collinear Points.

The exercises are very numerous and well chosen, and though the formal course deals only with plane geometry, there are many exercises involving three dimensions; but there does not appear to be any reference to *plan* and *elevation*. The letterpress and figures are well done, and very few errors are noticeable. On page 64 one of the seven measurements required to determine a pentagon has been omitted; the figure at the bottom of page 69 will scarcely be understood; the proof of Prop. 33 is incomplete.

It would improve the book to state explicitly the propositions of Solid Geometry which are postulated.

The Elements of Plane Geometry. By C. DAVISON. Pp. 280. 10s. net. 1920. (Cambridge University Press.)

This volume is printed in the elegant style of the Cambridge University Press, mainly for the sake of the numerous sets of exercises, each proposition being followed by a set of easy exercises which are direct examples on the proposition, while harder sets are placed at the ends of the eight chapters. There does not appear to be much that is new or original in the book-work. The first few propositions are Euclid I. 13, 15, 4, 16, 26, 5, 18, 6, 19, 8, 14, in this order. Proportion is postponed until Chap. VII. is reached, and the term *locus* is not introduced before page 245;—the double aspect, inclusive and exclusive, is, however, very clearly and explicitly expounded.

Exercises from Elementary Algebra. By C. GODFREY and A. W. SIDDONS. Vols. I. and II. complete with Answers. Pp. x + 395 + c. 7s. 6d. Without Answers 7s., or Vols. I., II. separately 4s. or 3s. 6d., 4s. 6d. or 4s. respectively. All prices net. 1920. (Cambridge University Press.)

With very few exceptions the Exercises are identical with those in the first edition of the well-known Elementary Algebra. The names of the compilers are sufficient guarantee of the value of these admirably graded sets of exercises and revision papers. W. J. DOBBS.

Euclid in Greek. Book I. With Introduction and Notes. By Sir T. L. Heath. Pp. viii + 239. 10s. net. 1920. (Cambridge University Press.)

Euclides Vindicatus. By G. Saccheri. Pp. xxx + 246. 10s. net. 1920. (Open Court Publication Co.)

(1) About thirty years ago the Trustees of the Johns Hopkins University consulted their Principal as to the choice of the professorial staff. His advice may be summed up in the pregnant words :—" Select a great mathematician and a distinguished Grecian." We doubt if, at the moment, President Gilman had in his mind any necessary relation between the subjects for the exposition of which two men of the finest capacity were to be found. Probably he intended to impress upon the mind of his Trustees that above all must the budding manhood within their gates be brought into the closest contact with the inspiration that is to be derived from the expression of the human spirit in the most exquisite of literatures, or from a discipline the value and intellectual interest of which is summed up in the famous warning inscribed on the portal of the school of Plato :—*μηδεις ἀγεωμέτρητος εἰσίτω μοῦ την στέγην*, and again in the master's words :—*ἀεὶ ὁ θεὸς γεωμέτρει*. To the science of geometry was given the name of a Greek. Euclid's *Elements* has reigned supreme for over twenty centuries, and has guided " that scientific thought which is one thing with the progress of man from a worse to a better state." As Clifford finely expresses it, " Far up on the great mountain of Truth, which all the sciences hope to scale, the foremost of that sacred sisterhood was seen, beckoning to the rest to follow her. And hence she was called in the dialect of the Pythagoreans, 'the purifier of the reasonable soul.' " It is curious that among the first of the " great " mathematicians to fill a chair at the University we have mentioned was one whose early experiences had made him a hater of Euclid. Sylvester's repugnance was based on different grounds from those which caused Newton for a short time to lay aside the *Elements* as " a trifling book." He wanted to see Euclid " honourably shelved or buried 'deeper than did ever plummet sound' out of the schoolboy's reach." De Morgan held that the study of Euclid was a better key to a greater quantity of useful reading than any other, and hoped that no other geometry would ever be studied in England. Cayley went so far as to prefer Simson's Euclid, and when reminded that this was not the pure gospel of Euclid, suggested—*the original text.*

Sir Henry Savile, who taught Greek and Mathematics to Queen Elizabeth, and who founded the Oxford Professorships in Geometry and Astronomy which bear his name, attached to them a library of which he wrote to Camden : " I have cleared my study of all the Mathematical Books, which I had gathered in so many years and countreys, Greek and Latin, printed and manuscripts, even to the very raw Notes that I have ever made in that argument, but with express charge, that they may make use of them, if there be anything of worth in them, but never to set out any of mine in print." It is odd to find that, though he lectured in public in Oxford on the *Elements*, he once obtained by the influence of Camden a loan of the Greek Text of Euclid from Sir Robert Cotton. It is, however, possible that of the seven editions existing at the time he lectured he may have desired to see a particular one for purposes of collation. His lectures carried his pupils no further than the first eight propositions of Book I., but his *Praelectiones tresdecim*, published in 1621, were delivered in the previous year, and it happens that in that very year Henry Briggs brought out the first six books in Greek with a Latin translation after Commandinus, " corrected in many places " (Heath, i. p. 102). Before sending out his lectures to the world, Savile may have wished to see the corrections made by one who, more familiar to us in connection with logarithms, was to be the first Savilian Professor of Geometry. Sir Thomas Heath describes the *Praelectiones* as " valuable because they grapple with the difficulties connected with the preliminary matter, the definitions, etc., and the tacit assumptions contained in the first propositions."

Since the days of Jack Cade the funeral of a grammarian has been productive of varied feelings. " Thou hast men about thee that usually talk of a noun and a verb, and such abominable words as no Christian can endure

to hear." But the traditional attachment to the dead languages persisted, and it is only within recent years that Greek "is not as compulsory as it was." With his usual facility in leaping to conclusions the man in the street may suppose that Greek and Euclid are both of them gone. Both are immortal. Greek will still be taught and still be loved where linguistic acquisitiveness impels, and where native genius instinctively embraces the Hellenic spirit.

It has occurred to Sir Thomas Heath, who is the greatest living authority on Greek mathematics, and whose monumental edition of Euclid is essential to all students of the book that has been declared to stand "pre-eminently at the head of all human productions," that it would be of no small interest and benefit to the higher Greek Forms could he enable them to "put themselves in the place of their fellow-students of twenty centuries ago." The result is the daintiest edition of the Greek text of the First Book that has yet appeared. The introductory matter opens with all that can be said about Euclid the man. Of the lives of the great Greek mathematicians we know next to nothing, for it is an extraordinary fact that there are no biographical details of, for instance, a Euclid, an Archimedes (except his being killed at the siege of Syracuse by the Romans in 212 B.C.), an Apollonius, or of a Diophantus.

The works of Euclid other than the *Elements* are enumerated, and the History of the *Elements* is given at some length. The sketch of the part played by Euclid in education passes over the ancient days when only the very old and exceptionally able men were allowed to study the mysteries of geometry, and begins with Athelhard of Bath and Gherard of Cremona. In the time of Roger Bacon, students at Oxford rarely advanced in the "craft of gemetry" further than the first four or five propositions, and it was not until the middle of the eighteenth century that Euclid became a school book in Britain. The Greek text is followed by notes, which are models of what notes should be, and an index of Greek terms explained in the notes. The little volume is of pocket-size, and—*experto crede*—stands wear well. We hope that after the appearance of his *magnum opus* Sir Thomas will be inspirited by the success of Book I. of *Euclid in Greek* to continue so promising a venture. We may soon expect to find teachers reviving Canon Wilson's example and setting "Jokes," of which the following is an example :—Solve, or turn into any other language, this problem : Find four points such that the line joining any two is perpendicular to the line joining the other two. We steal from its place among other "felicities" in the *Times Literary Supplement* (Sept. 30, 1920, p. 636), Arthur Sidgwick's fair copy :

> *στιγμὰς ἂν εὕροις τέσσαρας τοίας θέσει,*
> *ὥστ', ἣν ἑλόμενος ἡντινοῦν ξυνωρίδα*
> *γραμμῇ ξυνάψῃς, κᾆτα τὼ λελειμμένα*
> *ζεύξῃς ὁμοίως, τοῖνδε τοῖν γράμμαιν ἀεὶ*
> *ὀρθαῖς γενέσθαι γωνίαισι συμβολήν.*

sent on a postcard.

(2) **GREELY SAVANT TO PUBLISH UNUSUAL MATHEMATICAL WORK.** TRANSLATES, INTERPRETS A WORK TWO CENTURIES OLD WHICH WHEN PUBLISHED SOON WILL FURNISH A SURPRISE TO MATHEMATICIANS.

The untiring labours of Prof. G. B. Halsted in translating the forerunners of non-Euclidean Geometry have culminated in this edition of Saccheri's excessively rare volume, of which the full title is *EUCLIDES | Ab omni naevo vindicatus | sive | Conatus Geometricus | quo stabiliuntur | Prima ipsa universae Geometriae Principia*—thus modernised :—*Euclid | freed of every fleck | or | A Geometric Endeavor in which are | established the Foundation | Principles of Universal | Geometry.* It is heralded in the American Press, as the above earthquake-like "caption" will show, by a heart-whole enthusiasm which is rarely extended in this country to books of a similar character. As long ago as 1894 the translation of Saccheri into English was begun by Prof. Halsted in the *American Mathematical Monthly.* It was also translated in 1895 into German by Engel and Stäckel. The Latin text faced by an English translation occupies about 240 pages. The translation can lay no claims to literary

quality, but it is, in the large number of instances in which we have tested it, literal and followed without difficulty. In the second paragraph of the preface Prof. Halsted states that " no one has remarked that the erudite and accurate Sir Thomas Heath, in his three-volumed masterpiece has listed Saccheri's *Euclides vindicatus* as a Latin edition of Euclid, calling Saccheri its editor, a mistake so gross it could never have been made by anyone who had been privileged to see Saccheri's diadem." We imagine that Sir Thomas is quite able to look after himself, but it is perhaps worth while to say that the *list* of Latin writers in which the name of Saccheri occurs is headed " Versions or Commentaries," and that there is no mention of an editor at all. The entry is :—" Saccheri's *Euclides ab omni naevo* . . . is important for his elaborate attempt to prove the parallel postulate, forming an important stage in the history of the development of non-Euclidean geometry." On p. 145 he does not call him the editor of a Euclid, but " the editor of *Euclides ab omni naevo* . . . (1733) famous in the history of non-Euclidean geometry." The error is thus reduced from one of grossness to the dimensions of a mere *lapsus calami*, and we shall indeed be surprised if one word to justify the charge can be found in the *many* and *lengthy* references made by Sir Thomas to the famous Jesuit. If two wrongs make a right, there is little ground of complaint on either side, for the indignity of calling Saccheri a mere editor of his own book may be balanced, perhaps, by the enthusiasm of Prof. Halsted for the *ipsissima verba* of Sir Thomas Heath. We live, unfortunately, in a world in which, as Lord Bowen would say, we are conscious of each other's imperfections. If Prof. Halsted will modify his charge, or substitute for " gross " a milder epithet, we on our part will meet him half way, and promise to say nothing more about the fact that, blinded by the rays of the diadem, he has (inadvertently, of course) " lifted " without acknowledgment from the " three-volumed masterpiece " a whole page and a half for his own introduction. But we must forgive Prof. Halsted if his devotion to Saccheri makes him impatient of any fancied slur on the author of the *Logica*, which was " the coconut out of whose eyes the palm was to shoot up, which rises high above the flat and circumscribed old world,"—on the inventor of " the beauteous body of a new geometry " which " flowered, mermaid-like, the latter portions somewhat fishy, but oh ! the elegant torso." Does Prof. Halsted perceive the implication?

But, to quote Saccheri's closing words :—*haec jam satis.*

Textile Mathematics. Part I. By T. Woodhouse and A. Brand. Pp. 122. 2s. 6d. 1920. (Blackie & Sons.)

The *raison d'être* of such a little manual as this is obvious enough. It is suitable as a textbook of elementary type for instruction in Continuation Schools and classes, for early technical courses, and for beginners in our textile centres who have to rely on their own unaided efforts for the mathematics that will aid them in their work. Nearly all the examples and exercises are therefore drawn from the every-day life of the cloth-mills, etc. The stages are :—Signs, definitions, brackets, notation symbolic ; Rectangles and squares, triangles, quadrilaterals, polygons, circles, arcs, chords, segments, and sectors ; Gear wheels ; Rectangular solids, relative density, specific gravity ; Prisms and cylinders, pyramid, cone, and sphere. It is very simply written, beautifully printed, the letters and numbers being large enough to lay no strain on the eyes. It deserves success.

The Theory of Relativity. By R. D. Carmichael. 2nd edition. Pp. 112. 8s. 6d. net. 1920. (Chapman & Hall, for J. Wiley & Sons.)

The essential difference between the two editions of this monograph is the final chapter, which is designed to give a compact account of the generalised theory. Its scope will be gathered from the table of section headings. Following a general summary of the previous 72 pages : transformations of space in four dimensions ; principle of equivalence ; general transformation of axes ; theory of tensors ; covariant differentiation ; the Riemann-Christoffel tensor ; Einstein's Law of Gravitation ; the motion of a particle ; three crucial phenomena ; the electro-magnetic equations ; general considerations.

The Essentials of Mental Measurement. By W. BROWN and G. H. THOMSON. Pp. x+216. 21s. net. 1921. (Cambridge University Press.)

This is a revised edition of Dr. Brown's book, with new material from the pen of Professor Thomson—material which is largely controversial and which deals with recent developments in the theory of correlation.

Tables of Physical and Chemical Contrasts and Some Mathematical Functions. 4th edition. By G. W. C. KAYE and T. H. LABY. Pp. 161. 14s. net. 1921. (Longmans, Green.)

The following are the alterations and additions: "Matter relating to the figure of the earth, the absolute determination of the acceleration of gravity, and more extensive tables of the relative value of that constant; chemical data have been recalculated, using the international atomic weights; seven hundred additions and alterations in the physical constants of chemical compounds; the published values of these constants have been critically examined, and what appears to be more accurate values for the chemical compounds included in these pages have been added; heat tables revised and amplified; tables of atomic numbers, spark-gap voltages, X-ray wave lengths and terrestrial magnetic constants."

Exercises in Arithmetic. Arranged in Two Courses. By A. E. LAYNG, M.A. Pp. x+230+31. 3s. 6d.; with Answers, 4s. 1920. (Mr. Murray.)

This is an excellently devised little collection of exercises for the average boy, forming an easy course followed by one more difficult, and concluding with a large number of revision papers graduated for use at the various stages of either course. The printing is clear and the figures are large. "Questions on the papering of rooms have been excluded from the sections on Areas, for these questions are practical only in appearance, and are therefore misleading. For obvious reasons Recurring Decimals and Discount are omitted."

Vocational Mathematics. By W. H. DOOLEY. Revised by A. RITCHIE-SCOTT. Pp. vii+311. 5s. net. 1920. (Heath.)

This volume originally appeared in the United States, and has been adapted for English students by the reviser, his work mainly consisting in the adaptation to British units and terminology. Since mathematics cannot be taught in the workshop, the only thing that can be done is to bring subject, terminology, and even the slang of the subject into the mathematical course provided for the young apprentice in the evening school. This little volume contains the material for pupils occupied during the day in carpentering and building —sheet and rod metal work, bolts, screws and rivets, shafts, pulleys and gearing, plumbing and hydraulics, steam engineering, electrical work, machinists, lathe materials, planers, shapers, and drilling machines. The first eighty pages are given up to a revision of arithmetic, mensuration, and the arts of reading a blue print and drawing to scale. The book is certainly worth the attention of teachers of correlated subjects in evening schools.

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84. One to us.—To change the subject. Those pessimistic people who have been bemoaning our fate in the Test matches can take heart of grace. Not all our laurels are leaving us, and it bucks one considerably to learn that Mr. T. F. Gaynor, a young inventor, has at last succeeded in squaring the circle. You see, there is life in the old dog yet. Squaring the circle is one of the most ancient quests in the world, and it is old England that has brought it off at last.—*Daily News*, Jan. 15, 1921.

In their publication, the *Inventors' Annual Review*, the Inventors' Union claim that Mr. T. F. Gaynor, one of their members, has squared the circle, a problem—making a square *exactly* equal in area to a given circle— that has puzzled mathematicians for more than 2000 years. (From a local paper.)—Per Mr. J. W. Lewis.

LONDON BRANCH.

PROGRAMME FOR 1921-22.

1921.

Oct. 15th. THE LONDON DAY TRAINING COLLEGE, SOUTHAMPTON ROW, at 3 p.m.
Discussion on "SOME POINTS CONCERNING THE TEACHING OF THE 'UNMATHEMATICAL' GIRL."

Nov. 16th. THE LONDON DAY TRAINING COLLEGE, at 5.15 p.m.
Paper on "APPROXIMATE INTEGRATION," by A. Buxton, M.A., Imperial College of Science.

Dec. 3rd. THE LONDON DAY TRAINING COLLEGE, at 3 p.m.
Paper on "COEFFICIENTS OF CORRELATION," by Professor T. P. Nunn, D.Sc., M.A., London Day Training College.

1922.

Feb. 4th. Annual Meeting at THE LONDON DAY TRAINING COLLEGE, at 3 p.m.
Presidential Address on "DIFFERENTIAL EQUATIONS IN MECHANICS AND PHYSICS," by Professor A. R. Forsyth, Imperial College of Science.

Mar. 11th. THE LONDON DAY TRAINING COLLEGE, at 3 p.m.
(*a*) AN INFORMAL TALK ON POINTS IN MATHEMATICAL CLASS TEACHING.
(*b*) Paper on "THE TENSOR NOTATION FOR DETERMINANT PROPERTIES AND FOR LINEAR RELATIONS," by W. F. Sheppard, Sc.D., M.A.

It is hoped to arrange a visit to the Meteorological Observatory at Kew, in June. The date of this visit will be settled later.

Secretaries: Miss L. A. Zelensky, B.Sc., Haberdashers' Aske's Girls' School, Acton, W. 3.
G. Goodwill, B.Sc., 37 Park Road, Dulwich, S.E. 21.

FOR SALE.

A complete set of the *Mathematical Gazette*, Nos. 1-144, in parts as issued. Price Five Guineas.—Apply: W. G. Bell, Calvert's Cross, Beaconsfield, Bucks.

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Reserves are kept of A.I.G.T. Reports and Gazettes, and, from time to time, orders come for sets of these. We are now unable to fulfil such orders for want of certain back numbers, which the Librarian will be glad to buy from any member who can spare them, or to exchange other back numbers for them:

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A.I.G.T. Report No. 11 (very important).
A.I.G.T. Reports, Nos. 10, 12.

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